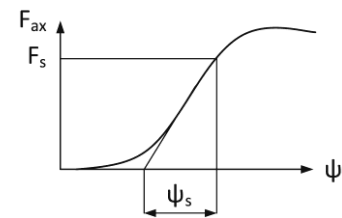


## Correct screw pretension

### Theory

Traditionally we tighten a bolted joint by turning bolt/screw or nut with the aim of reaching an axial force  $F_{ax}$  in the bolt/screw of 85-90% of yield force  $F_s$ . In modern screw/bolted joints we often tighten to full plasticity. For a class 8.8 screw onset of yield corresponds to a strain of

$$\epsilon_s = \frac{\Delta L}{L} = \frac{\sigma_s}{E} = \frac{640}{210} \cdot 10^{-3} = 0,00305$$



Through experiments we know that  $F_{ax}(\psi)$  behaves as shown in the figure.

First there is an elastic deformation removing play in the joint. Then  $F_{ax}$  increases linearly until onset of yield

$F_s$ . With compressed parts taken to be rigid, the angle  $\psi$  will be underestimated because in a real joint the clamped part, being elastic, will be compressed while the screw is elongated. Further, there is deformation of screw head, nut, and threads.

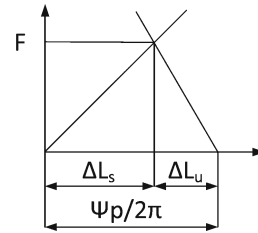
The screw is elongated  $\Delta L_s$  and the clamped joint is compressed  $\Delta L_u$ . These deformations are caused by pitch  $p$  and by the turning of screw/nut angle  $\psi$ . We have

$$\psi \frac{p}{2\pi} = \Delta L_s + \Delta L_u$$

From the figure

$$F = k_s \Delta L_s = k_u \Delta L_u$$

$$\Delta L_u = \frac{k_s}{k_u} \Delta L_s$$



which yields

$$\psi \frac{p}{2\pi} = \Delta L_s + \frac{k_s}{k_u} \Delta L_s = \left(1 + \frac{k_s}{k_u}\right) \Delta L_s, \quad \Delta L = \epsilon L$$

$$\psi_s = \frac{2\pi}{p} \left(1 + \frac{k_s}{k_u}\right) L \epsilon_s$$

**Screw.** We now determine  $k_s$  and  $k_u$ . To include the elasticity of the screw head, threaded part of goods or nut, take  $L_s > L$ . The size of the addition grows inversely progressive to  $L/d$  and  $R/d$ . A typical estimate is given in the table.

Type of head/nut:	Hex	Allen	Flange	
Clamped part	steel	steel	steel	aluminium
Head	0.74d	0.78d	0.82d	1.11d
Thread (screw & nut)	0.67d	0.67d	0.78d	0.78d
Nut	0.54d	0.54d	0.49d	0.65d
Sum	1.95d	1.99d	2.09d	2.54d

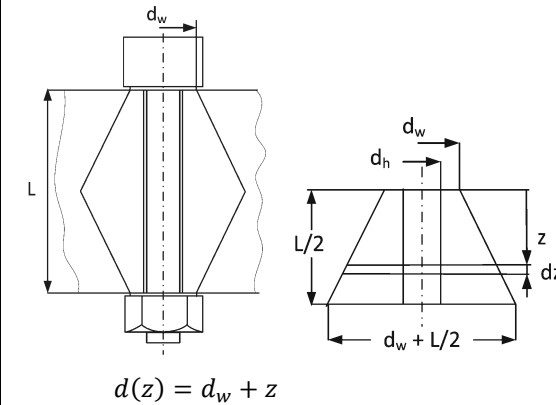
Screw length can be taken to be clamped length  $L$ +addition for head, etc.

$$L_s = L + 1.95d, \text{ for a hex screw and nut}$$

$$k_s = \frac{E_s A}{L_s} = \frac{E_s \pi d^2}{4L_s}$$

If a screw has sections with different diameters, then the stiffness/spring constant  $k_i$  for each section should be calculated separately. The total spring constant is  $k_s = (\sum k_i^{-1})^{-1}$ .

**Clamped part** can be approximated by a truncated double cone with a cylindrical hole. Cone angle  $\varphi$  is such that  $\tan \varphi = 0,5$  (Birger-cone). This approximation yields results that are in good agreement with FEA-analysis of bolted joints.



$$d(z) = d_w + z$$

$$\text{Disc area } A(z) = \frac{\pi}{4} (d_w + z)^2$$

Stiffness are summed

$$\frac{1}{k'} = \int_0^{L/2} \frac{dz}{EA(z)} = \frac{4}{\pi E} \int_0^{L/2} \frac{dz}{(d_w + z)^2} = \frac{4}{\pi E} \left[ \frac{-1}{d_w + z} \right]_0^{L/2} = \frac{4}{\pi E d_w (d_w + L/2)}$$

which yields for a truncated cone

$$k' = \frac{\pi E}{2L} d_w \left( d_w + \frac{L}{2} \right)$$

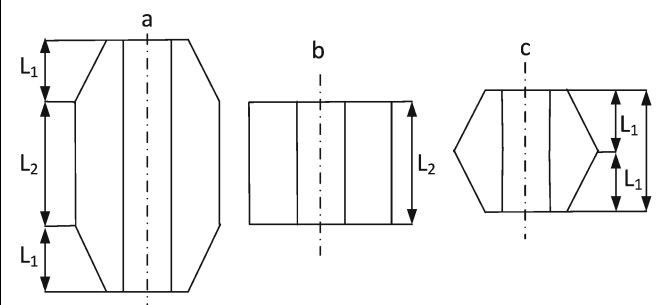
For a double cone the stiffness is halved (serially coupled stiffness)

$$k = \frac{1}{2} k' = \frac{\pi E}{4L} d_w \left( d_w + \frac{L}{2} \right)$$

And with reduction due to the hole cylinder (parallel stiffness)

$$k_u = \frac{\pi E}{4L} \left[ d_w \left( d_w + \frac{L}{2} \right) - d_h^2 \right]$$

If there is not enough material (less than  $d_w + \frac{L}{2}$ ) around the screw, the stressed volume will be a truncated double cone + a cylinder. Both with a coaxial cylindrical hole.



$$\text{Here } k_b = \frac{\pi E}{4L_2} (D_a^2 - d_h^2)$$

$$k_c = \frac{\pi E}{4L} \left( d_w \left( d_w + \frac{L}{2} \right) - d_h^2 \right)$$

Summing yields the spring constant

$$k_a = \left( \frac{1}{k_b} + \frac{1}{k_c} \right)^{-1}$$

### Method

*Comment:* Flange screw  $d_w = 1.9d$ , flange nut  $d_w = 2d$ . Hex head & nut  $d_w = 1.45$  and  $1.6d$  for Allen head.

1. Note data  $p, d_w, L_s, E_s, d, d_h, L, E_u$
2. Is  $d_w + \frac{L}{2}$  larger than the width of the joint? Does the double cone fit within the joint?
3. Calculate  $k_s$  and  $k_u$

*Alternative to points 2 & 3:* read  $\frac{k_s}{k_u}$  from curves 1 or 2.

4. Calculate  $\psi_s = \frac{2\pi}{p} \left(1 + \frac{k_s}{k_u}\right) L \epsilon_s$  ( $\epsilon_s = 0,00305$ )
5. Angle is in radians. Convert to hex sections

$$m = \frac{\psi_s \cdot 180}{60\pi} = \psi_s \frac{3}{\pi}$$

6. Tighten hard to remove play
7. Loosen screw/nut
8. Tighten to remove play ("finger tight")
9. Turn "number of hex sections",  $m$ .

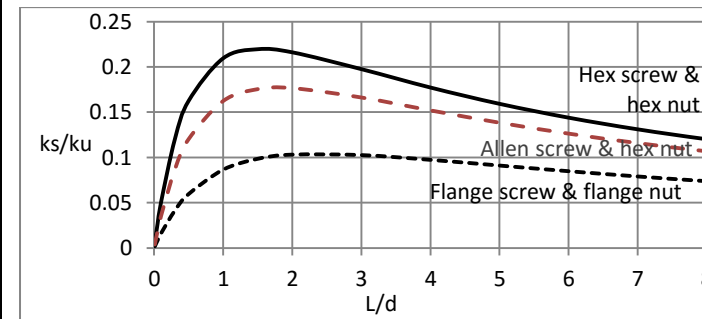
### Comments

We see by the formula  $\psi_s = \frac{2\pi}{p} \left(1 + \frac{k_s}{k_u}\right) L \epsilon_s$  that the extra turn of the screw due to elasticity of the clamped part, depends on  $\frac{k_s}{k_u}$ .

With  $d_h = d$ ,  $d_w = \theta d$ ,  $L = nd$ ,  $L_s = L + \xi d$ ,  $d_h = \zeta d$ ,  $E_s = E_u$  we get

$$\frac{k_s}{k_u} = \left[ \left(1 + \frac{\xi}{n}\right) \left[ \theta \left( \theta + \frac{n}{2} \right) - \zeta^2 \right] \right]^{-1}$$

The factor  $\frac{k_s}{k_u}$  is shown in the figure below.



### FEA calculation

The curves below show FEA calculations of bolted joints for different  $L/d$  and  $R/d$ , where  $R$  is the radius from screw centerline to the end of clamped material. Same material in all parts and screw hole is  $1.1d$ .

Conditions in the thread area are very complicated and have been extensively simplified.

Diagram 1

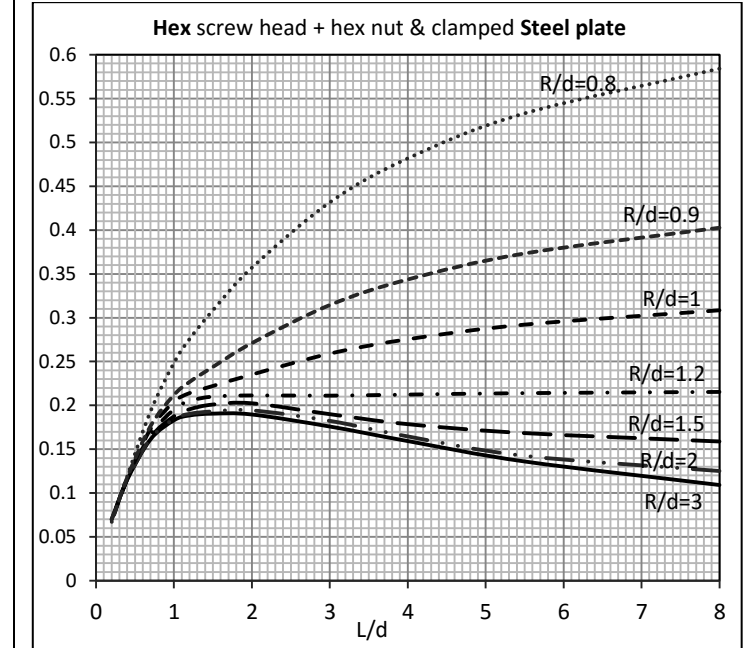
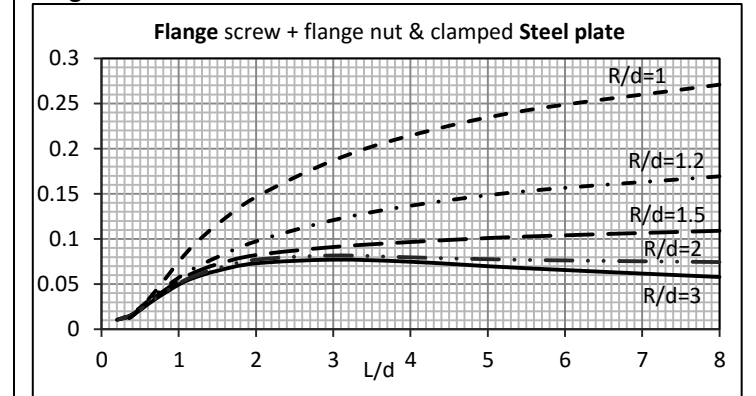


Diagram 2



### Commentaries

Screw in threaded hole differs marginally from screw in nut if  $L$  is taken from head to first engaged thread and  $L/d > 1$ .

The elastic deformation of the screw head is surprisingly large. If Young's modulus of the clamped part is reduced, e.g. Al instead of steel (70GPa instead of 210GPa) deformation of the clamped part increase strongly,  $k_s/k_u$  will be nearly three times larger, and the deformation of the head is increased.

Screw stem diameter can be from 3-5% (M6) to 1.7-2.6% (M20) less than nominal diameter, depending on production class.

The thread must be well lubricated, preferably with a solid lubricant.